New Design Ideas
Vol.4, No.2, 2020, pp.69-84

A BRIEF REVIEW ON FRACTALS, THEIR APPLICATIONS AND IMPLICATIONS

Luís Borda-de-Água[1,2*]

[1]CIBIO/InBio, Centro de Investigação em Biodiversidade e Recursos Genéticos, Laboratório Associado, Universidade do Porto, Campus Agrário de Vairão, 4485-661 Vairão, Portugal
[2]CIBIO/InBio, Centro de Investigação em Biodiversidade e Recursos Genéticos, Laboratório Associado, Instituto Superior de Agronomia, Universidade de Lisboa, Tapada da Ajuda, 1349-017 Lisbon, Portugal

Abstract. Fractals (natural or man-made) are objects possessing detail at all scales. One of their attractions is the ability to reproduce natural objects, from mountains to cellular structures, but fractals have also been founded in several human artefacts, such as, in decorative geometric patterns, in architectural strucutres and in the layout of cities. Therefore, it is not surprising that the study of fractals has become mainstream in science and engineering, but that their applications have also been extended to the social sciences, music and architecture. In architecture, fractals and related tools can be used in two ways. On the one hand, fractals can be a feature of the architectural design, on the other hand, fractal methods can be used to the characterization of existing structures. The main purpose of this article is to provide a brief introduction to fractals, with an emphasis on the distinction between different categories (self-affine versus self-similar and deterministic versus statistical) while, simultaneously, providing an overview of the application of fractals in different fields. One of the implications of fractal studies, partially because of their ability to unify disparage disciplines, is that we have now the tools to start understanding the characteristics of the environments that best adapt to our needs. I end with a brief note on fractals and modern architecture, and how fractals alone are not enough to guarantee the desirable qualities of good architectural design.

Keywords: *Fractal dimension, Interdisciplinarity, Self-affinity, Self-similarity.*

[*]***Corresponding Author:*** *Luís Borda-de-Água, CIBIO/InBio, Centro de Investigação em Biodiversidade e Recursos Genéticos, Laboratório Associado, Universidade do Porto, Campus Agrário de Vairão, 4485-661 Vairão, Portugal, mail: lbagua@gmail.com*

Received: *20 July 2020;* ***Accepted:*** *24 August 2020;* ***Published:*** *30 December 2020.*

1. Introduction

Fractals are everywhere and fractal geometry can connect disparate disciplines. Fractals open a new vision and understanding of the world, not only of natural phenomena but also of man-made objects and phenomena. Here I review different categories of fractals, and how fractals have emerged in a wide variety of disciplines, from history to music and painting, and, of course, architecture, and, more importantly, how fractals unify these disciplines.

Fractal geometry is an extension of, but also a departure from, traditional Euclidean geometry. Thanks to fractal geometry we came to realize the inadequacy of Euclidean geometry to deal in a satisfactory way with some natural phenomena or even man-made objects. As Mandelbrot (1983), the father of fractals, put it: “Clouds are not spheres, mountains are not cones, coastlines are not circles, and bark is not smooth, nor does lightning travel in a straight line”. What the objects in this now famous sentence have in common is that they are not "smooth", they are characterized by shapes that are "grainy, hydralike, in between, pimply, pocky, ramified, strange, tangled, tortuous, wiggly, wispy, wrinkled" (Mandelbrot, 1983). The objective of fractal geometry is to provide tools to deal with the "roughness" of real objects and phenomena that until recently could only be dealt with in a rather contrived way because of the limitations of traditional geometry. Not surprisingly, one the most publicized characteristic of fractals is their ability to accurately reproduce a wide range of natural objects.

It would be a mistake to assume that fractal geometry has only found a role in describing physical "palpable" objects. Fractals also been important in revealing features of a wide variety of phenomena, such as, to show the commonalities of different musical genera (e.g., Hsü, 1993), to quantify the perceived visual differences among the works of artists such as Pollock (e.g., Taylor *et al*., 1999) or between buildings of different architects, such as, those by Frank Lloyd Wright and Le Corbusier (e.g., Bovill, 1996). The realization that fractals structures are present in both natural and man-made structures has had important implications, in particular with regard to how we perceive our surrounding environments. In this respect, experiments in psychology have revealed that images with fractal characteristics helped reduce the stress of the participants when these had to perform demanding tasks (e.g., Taylor 2006).

One of the aims of this review is to provide an introduction to fractals, to show how they can be generated and how they can be measured. This review should be seen as a guide that can be used to interpret fractal-like objects. Accordingly, I will describe the following categories of fractals: first, self-similar and self-affine and, second, deterministic and statistical fractals. I chose these categories because they are the ones I use to describe in a first qualitative way an object that I hint to have fractal attributes. I emphasize this is not a mathematical review on fractals, but I hope it provides an entry point for those interested in fractals before moving on to more specialized literature.

2. What are fractals?

Interestingly, Mandelbrot, the father of fractal geometry, was reluctant to provide a precise definition of fractals. A non-mathematical definition by Mandelbrot (1975) is: "Fractals are mathematical objects, whether naturally or human made, which can be described as irregular, coarse, porous or fragmented, and which, furthermore, possess these properties to the same extent on all scales." Here too, I will not give a precise definition of fractals, instead, I describe different categories of fractals that according to my experience are the most relevant to identity and interpret natural and man-made fractals.

Two important categories are the self-similar and self-affine fractals. The self-similar fractals are the most famous ones; Fig. 1 provides an example, called the Sierpinski carpet. Typically, to obtain a fractal there is an algorithmic process that evolves repetition. For example, to obtain the Sierpinski carpet apply the following algorithm: (i) start with a square and divided it into 9 squares, (ii) remove the central square, (iii) repeat the above two rules to all remaining squares, and (iv) keep iterating these procedures the desire number of times. In the idealized mathematical version these iterations are done an infinite number of times. The salient point here is that the repetitive process leads to an important feature of fractals: *the existence of detail at all scales*. In the case of self-similar fractals, the construction process is such that parts of the fractal object are identical at different scales. When this happens, the size of a portion of the fractal cannot be determined unless there is a reference to which it can be compared. This makes these fractals ideal to perform (willing or unwillingly) visual tricks. In order to avoid such tricks, and because geological objects, such as rocks, tend to exhibit fractal characteristics, geologists usually include in a photograph an object of known size, such as a hammer or a compass, to ensure that the size of the geological structure can be approximately determined (e.g., Holmes 1965). Notice that this practice long predates the formal introduction of fractals, revealing that the notion of self-similarity has long been perceived.

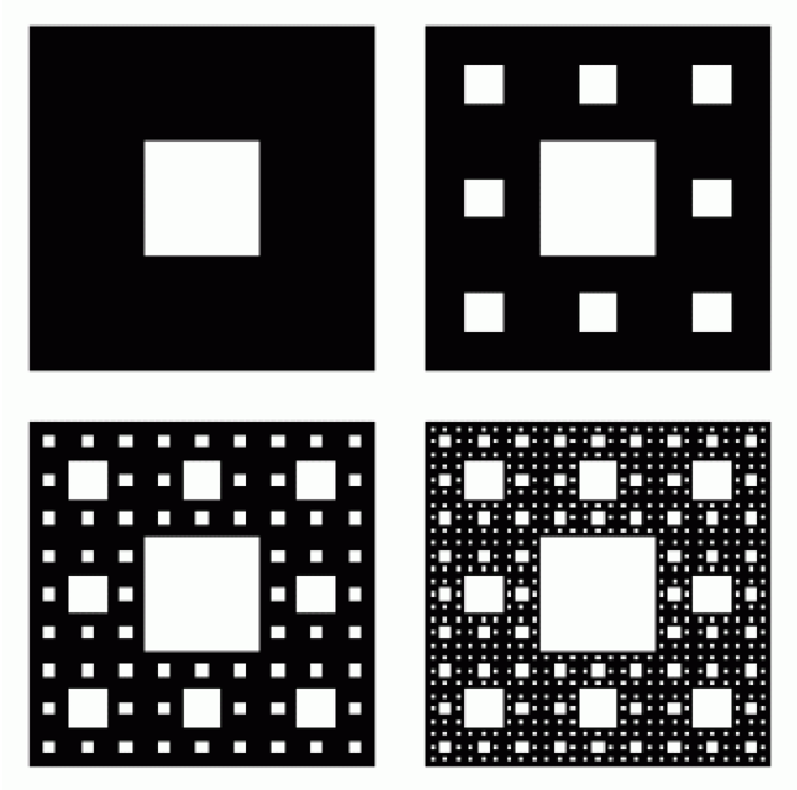

Figure 1. The first four iterations of the Sierpinski carpet, a self-similar fractal

In contrast to self-similar fractals, self-affine fractals appear different depending on the scale at which they are observed. A good example is a range of mountains, or the silhouette of a city like New York, at least the borough of Manhattan. When seen from far way, a mountain range looks almost like a straight line in the horizon, but when one gets closer the different peaks becomes more noticeable. Similarly, from far way Manhattan looks a relatively flat line, but the difference among the heights of the buildings becomes more obvious when viewed more closely. Figure 2 provides an example that one can thing of as a mountain chain that is being approached when moving from the plot a to plot c. However, to understand the construction of this self-affine fractal one should progress from plot c to plot a. Plot c is the “generator” of the fractal. This generator is made of 4 segments of equal size. In order to obtain plot b substitute each of the four segments in plot c by a smaller replica of the generator. Finally, to obtain plot a we substituted each of the 16 segments in plot b by an even smaller version of the generator. (Of course, one could continue this iteration process indefinitely.) What characterizes the construction of this fractal is that it is "stretched" differently along the vertical and horizontal directions, in contrast to a self-similar fractal where both directions are treated equally. In a sense, self-similar fractals are just a particular case of self-affine fractals where the "stretching" is equal in both directions.

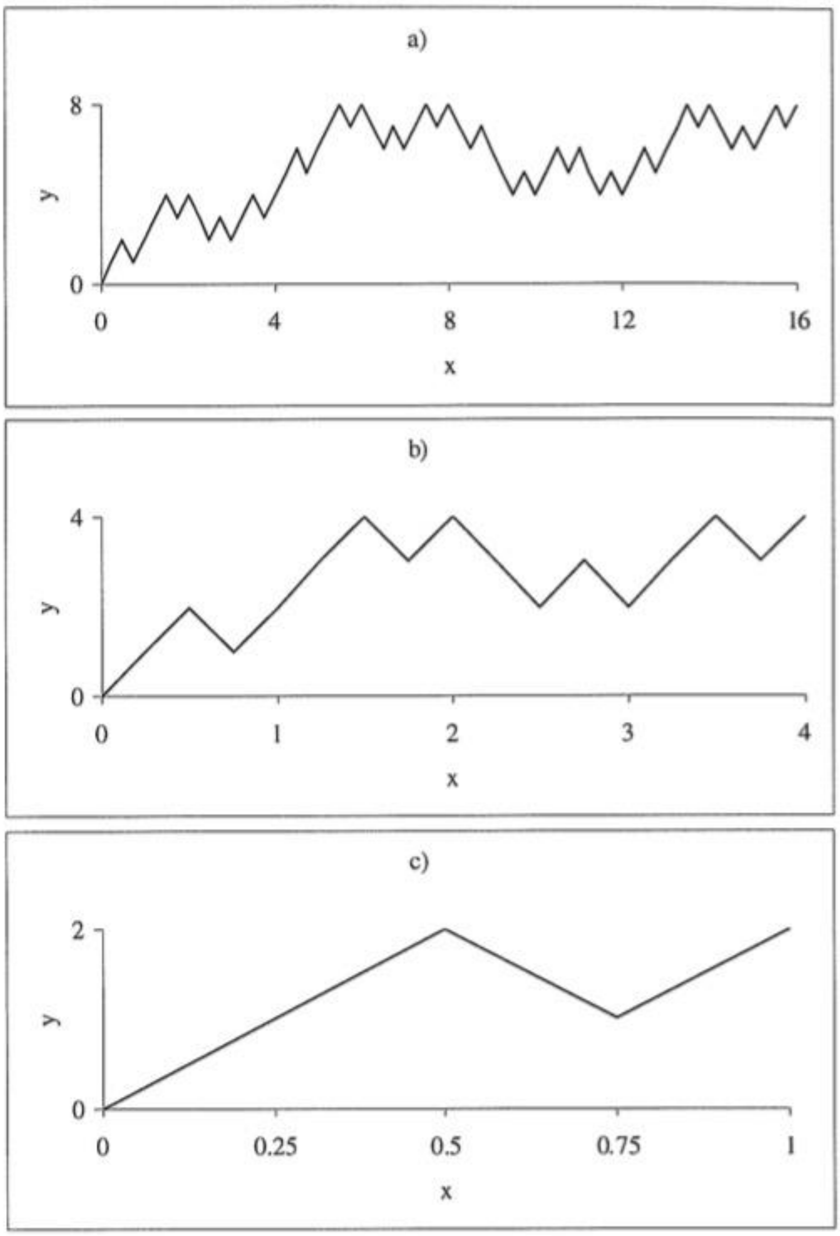

Figure 2. Example of a self-affine fractal. Imagine these figures as a mountain being seen progressively closer when moving from the top to the bottom plot. However, to understand the construction it is easier to progress from plot c to plot a

Another important distinction is between deterministic and statistical fractals. Deterministic fractals are obtained when we repeatedly iterate a rule without allowing chance events to occur. For instance, the Sierpinski carpet, Fig. 1, and the self-affine "mountain", Fig. 2, are deterministic fractals. Statistical fractals are obtained when we introduce randomness in the process of the fractal formation. Consider the curves on the left-hand side of Fig. 3; these shows the first stages of the (fractal) Koch curve, still a deterministic fractal. Its rules of formation are: (i) start with a segment and divide it into 3 segments (top line of Fig. 3a), (ii) remove the central segment and substitute it by two other segments forming a triangle (second line on Fig. 3a), (iii) keep applying the two previous rules to the remaining segments. In order to obtain the equivalent statistical version, we add chance events using, for instance, a coin. Let us assume that "heads" means the "triangle" substituting the removed segment is pointing "up", and "tails" implies the "triangle" is pointing "down" (I'm using inverted commas for "up" and "down" because after a few ietrations up and down are no longer the best way to describe where the triangle is pointing to). As before, we started with a segment, divided it into 3 segments and removed the central one, but now we flipped a coin in order to define if the substituting triangle is pointing up or down. The result was "heads", so the triangle is pointing up (in this case, at this iteration level we obtained exactly the same curve as in the deterministic case). In the next stage, and as before, we divided the four remaining segments into three, but flipped a coin four times in order to determine the direction of the triangles. We obtained "tail", "head", "tail" and "tail", hence the substituting triangles are pointing "down", "up", "down" and "down", the third line from the top in Fig. 3b. The other lines in Fig. 3b show the result of applying the same procedure at other stages of the Koch curve. Compare the bottom lines of Fig. 3a and 3b: while the deterministic fractal shows clearly the algorithm used to obtain it, the statistical one looks more "natural", closer to what one could expect from a natural object, such as the contour of a cloud. Incidentally, this capability of fractals to reproduce natural objects has not escaped Hollywood's attention, which soon began to use fractal techniques in filmmaking (e.g., Barnsley *et al*., 1988, or Peitgen & Saupe, 1988).

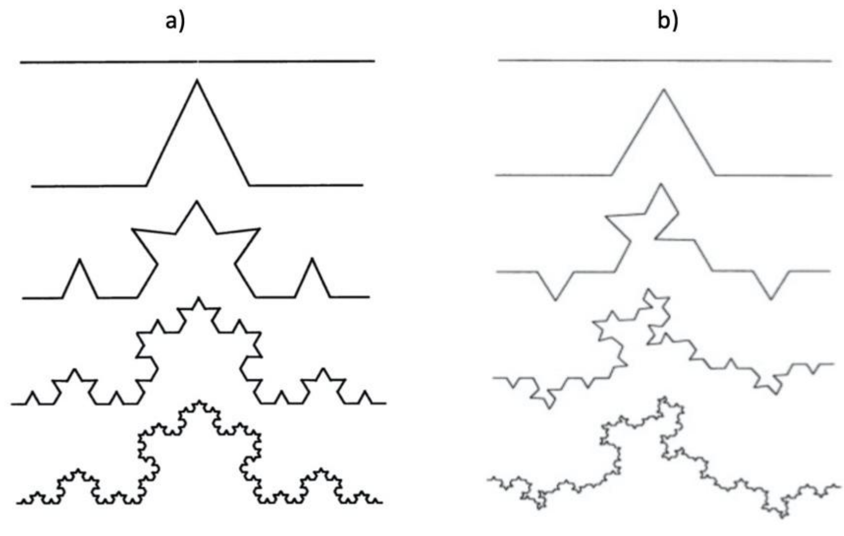

Figure 3. The Koch curve, on the left hand side, the first four stages of the deterministic version, and on the right hand side the four stages of a concretization of the random (statistical) version

Often, fractals are not enough to describe an object. In fact, most real objects are multifractals. In these cases, an object is not defined solely by one fractal dimension, but by a spectrum of fractal dimensions (e.g., Borda-de-Água *et al.*, 2007). The basic difference between fractals and multifractals is that the former pay attention only to the presence or absence of a given feature, while the latter consider, in addition, the relative amount of that feature. For example, to obtain the (fractal) Sierpinski carpet we simply removed squares, thus the final structure is black and white. On the other hand, in a multifractal Sierpinski carpet the squares also have associated a "weight". To obtain a deterministic multifractal we perform what is called a "multiplicative cascade". For instance, to obtain a multifractal Sierpinski carpet we start with a "generator" with weights (such as numbers between 0 and 1) assigned to each square. We can imagine that these weights represent different shades of grey as in Fig. 4a. Notice that because the central square of the generator has weight zero, the entire structure is still like the Sierpinski carpet. In each iteration the values in each of the squares are multiplied by the "generator". For example, the first iteration corresponds to the generator itself. Then, in the second iteration the top left square, that has a value of 0.07, is multiplied by the generator, resulting into nine squares whose values (and grey shades) corresponding to 0.07 times the values defined in the generator. Equally, the central square on the top row, with value 0.05, is multiplied by the generator leading to another nine squares with values (and grey shades) corresponding to 0.05 times the values defined in the generator. This process is repeated to all remaining squares finishing the second iteration. We can now re-start this procedure to get the third iteration, *et seq*. The resulting multifractal after four iterations is shown in Fig. 4b.

a

b

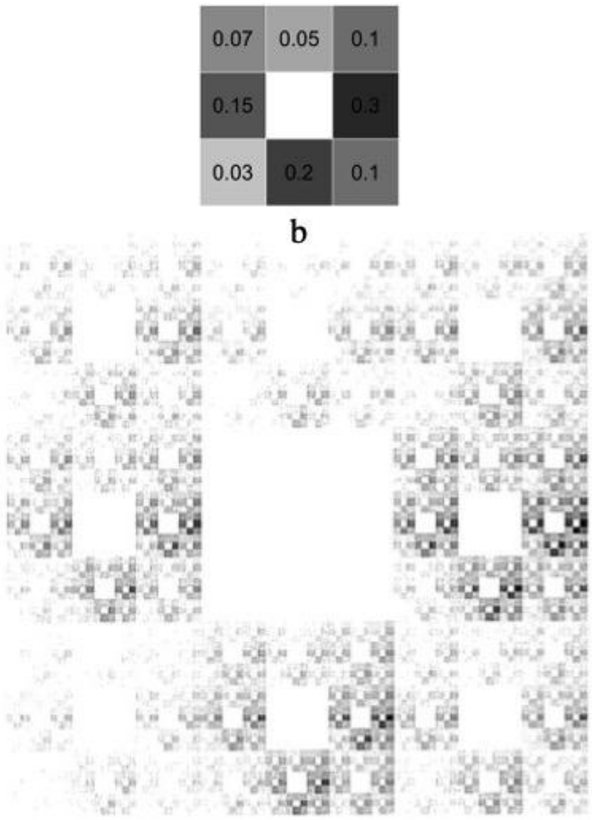

Figure 4. Example of multifractal using the Sierpinski carpet as support. Plot (a) shows the generator, where the numbers correspond to the weights assign to each square and the darker and lighter regions correspond to higher and lower weights, respectively. Plot (b) shows the resulting Sierpinski multifractal carpet after 4 iterations – compare it with the Sierpinski fractal of Fig. 1.

3. Quantifying fractals

The above description of fractals was merely qualitative. However, fractals can be characterized quantitatively by extending the notion of dimension to non-integer numbers. Such non-integer dimensions are called fractal dimensions, and an important part of the literature on fractals is on the development of methods to estimate fractal dimensions. A very simple method is called box-counting and, to the best of my knowledge, it is the only one that has been used to characterize architectural features, as we will see later.

The concept of a non-integer dimension may sound strange at first. To obtain an intuitive understanding of a non-integer dimension, consider a line in a surface. In Euclidean geometry a line has (always) dimension 1. In fractal geometry a line embedded in a surface has a fractal dimension between 1 and 2, depending on how it "occupies" the surface: "1" corresponds to a smooth line (straight or curved) and "2" to the limit of a line so compact and convoluted that it becomes a surface. Identically, in Euclidean geometry a surface has (always) dimension "2", but in fractal geometry a corrugated surface has fractal dimension between 2 and 3, the latter being a surface that completely fills the space.

The proper mathematical definition of fractal dimension involves the concept of a limit to zero or to infinity, depending on how we set the scaling factor (e.g., Mandelbrot, 1975, 1983). In this sense, real objects are never truly fractal, one can only expect that they are approximately fractal within a range of scales, from where we can estimate the (approximated) fractal dimension. As mentioned before, one approach to estimate the fractal dimension is the "box counting" method. In two dimensions it consists of covering an object with squares ("boxes") of different sizes, count how many squares are required to cover the object, and then repeat this process for squares of different size. For example, assume that the object under study is a set of points, as in Figure 5. To apply the box-counting method, start with a mesh of a given size (plot 5a), cover the points and count who many squares contain at least on point; in this example it is 19 squares, as shown in plot 5b. As always with fractals, there is a repetitive process applied at different scales. Accordingly, the next steps of the box counting method consist of repeating the process of covering the points with meshes of different square sizes, and for each mesh size count how many squares contain at least one point. Once this has been done at different scales, the number of occupied squares is plotted as a function of their linear size (the length of the side of a square) in a double logarithmic plot. If the object is a fractal, a straight line should emerge, and the slope of this line is equal to the negative value of the (box-counting) fractal dimension, plot 5c. (Fig. 7 below provides an example of the application of the box-counting method to the front elevation of two buildings.) Although simple to apply, box-counting is rarely practical, because for very large boxes all the squares are occupied, leading to a dimension equal to 2 (that of a surface), and for very small boxes the number of occupied squares remains constant and equal to the number of points; the implication of this is that the range of values from where the fractal dimension can be estimated tends to be very small.

For multifractals instead of one fractal dimension there is a "spectrum of dimensions". To calculate the spectrum for the distribution of points of Fig. 5 we would need to consider the proportion of points falling into each square and not merely their presence or absence. For more details and for alternative methods to estimate the fractal dimension see, e.g., Borda-de-Água et al. (2007). In the section "Fractals everywhere" I will describe another very simple method to estimate the fractal dimension of a line.

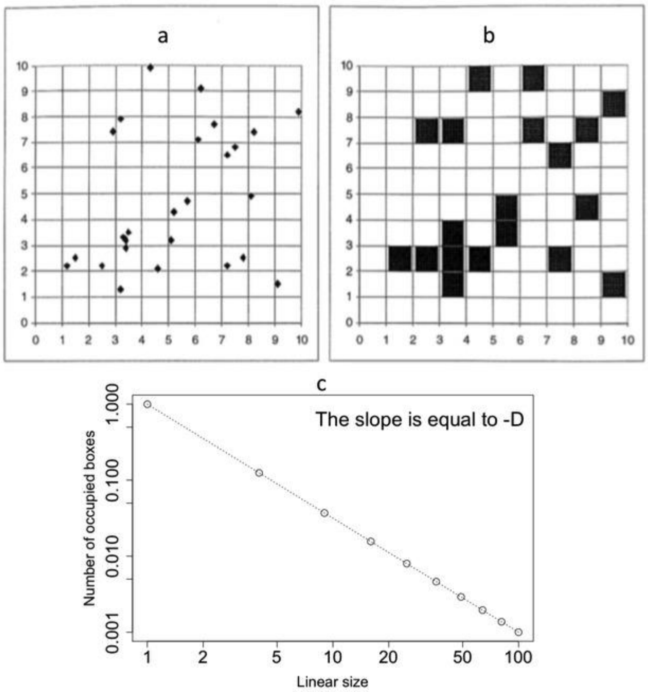

Figure 5. The box-counting method. Plot a shows the mesh of squares used to cover the set of points and plot b shows the squares that contain at least one point. Plot c illustrates the (idealized) straight line after the application of the box-counting method, from where the slope of the line and then the (box-counting) fractal dimension, D, is estimated

4. Fractals Everywhere

The main purpose of this section is to review the application of fractals to a wide range of scientific disciplines and the insights that studying fractals have brought to the understanding of human psychology. The title of this section is an allusion to the book by Barnsley (1988), but interpreting here "everywhere" not only as fractals being physically present everywhere but as well as a phenomenon present in a wide variety of disciplines. We start with an example that has, at least partially, roots in history. (Dotsenko (2020) provides other examples of applications of fractals.)

History. A now classic example of a fractal is the length of the border between Portugal and Spain. Richardson (1961) observed a rather different estimate depending on whether the source was Portuguese or Spanish: while the Portuguese claimed a size of 1,214 km, the Spanish mentioned only 987 km. This difference was likely due to the choice of the size of the "rulers" used to estimate the border's length, as Fig. 6a illustrates. The Portuguese, by choosing a smaller sized ruler, could follow more closely the real border and, hence, obtained a larger length; I decline to speculate why the Portuguese and the Spanish chose "rulers" of different sizes. Furthermore, Richardson noticed that if other measurements based on other rulers' size were added, they would fall approximately on a straight line when plotted a log-log graphic, Fig. 6b. This example was later picked up by Mandelbrot (1983), who pointed out that the emergent straight line in a log-log plot is what one should expect from a fractal curve, and that the slope of the straight line is an estimate of the fractal dimension.

Often, the border between two countries is a river, and rivers meanders are known to have fractal geometries, but in other cases, even when there are no clear natural boundaries, borders can be very convoluted, as in the case of the border between Portugal and Spain, reflecting the turbulent history between the two countries. Compare the shape of Portuguese-Spanish border with the perfect straight line of the west portion of the border between the United States and Canada, which resulted from a peaceful agreement between the two countries.

Incidentally, this example also helps show that even when measures at different scales exhibit different values, an underlying invariance can still be found, and it is this invariance that we are often interested in; see Borda-de-Água (2019) for more details. In the above case, rulers of different sizes led to borders of different lengths, however, when these lengths were plotted in double logarithmic scales a straight line emerged, implying a constant slope, from where the fractal dimension (the invariance) was estimated.

a b

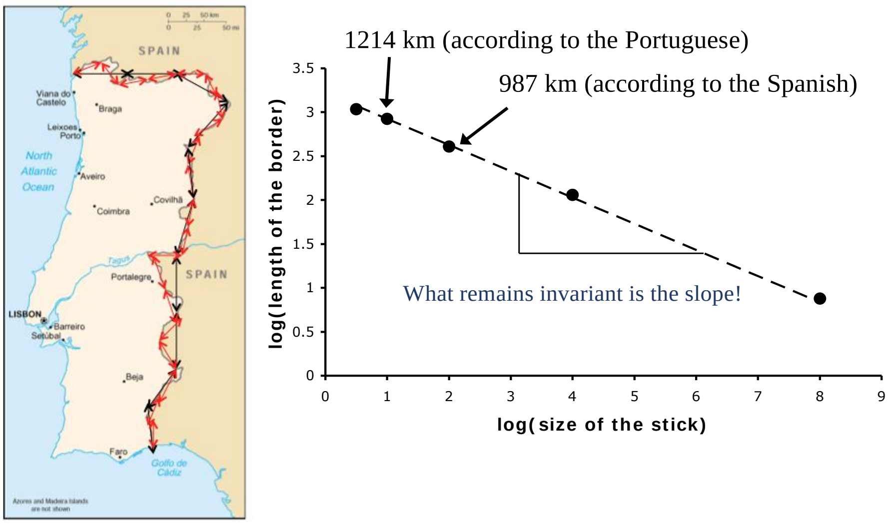

Figure 6. An example of measuring the border between Portugal and Spain using rulers of two different sizes (plot a), and the line formed by plotting several measurements in a log-log graphic (plot b)

Music. Voss and Clarke (1975, 1978) showed that the power spectrum of the loudness of very different musical styles, from Bach's Brandenburg Concertos to jazz, blues and rock, showed a distinct distribution that is typical of fractals. This topic was further developed by Hsü (1993) who pointed out that even bird songs display fractal like patterns. In contrast, Hsü (1993) showed that Stockhausen's Capricorn (1977), a modern music composition, did not exhibit the typical distribution of fractals.

Painting. Taylor et al. (1999) showed that the fractal dimension of Jackson Pollock's drip paintings increased consistently over time, going from a fractal dimension of approximately 1 in 1943 to approximately 1.72 in 1952. Kim et al. (2014) analysed the mathematical patterns of paintings of several historical periods of western paintings from the Middle Ages to the 20th century, and found a consistent fractal pattern, but with historical periods characterized by different fractal dimensions; for example, Pollock's drip paintings had a fractal dimension similar to that of the Middle Ages paintings but significantly inferior to that of the other historical periods analysed.

Architecture and urbanism. In the field of urbanism Batty and Longley (1994, page 2) pointed out that "[p]lanned cities are cast in the geometry of Euclid but by far the majority, those that are unplanned or planned less, show no simplicity of form". More recently, Chen (2009) showed that cities are, in fact, self-affine fractals, revealing different growths along different directions. In a work with also sociological implications, Wong et al. (1999) used fractals methods to describe the spatial segregation (e.g., racial) in 30 US cities.

Fractal-like patterns have been used in the design of buildings and other human artefacts (Bovill 1996). That being the case, one can assume that fractals can be used to characterize buildings of different styles and periods. For example, Bovill (1996) and Ostwald et al. (2015) analysed the the front elevation of the Robie house (Frank Lloyd Wright) and of the Villa Savoye (Le Corbusier) and showed that the former has a higher fractal dimension than the latter. Fig. 7 shows the application of the box-counting method at different scales of resolution to the estimation of the fractal dimensions of works by Frank Lloyd Wright and Le Corbusier (but see below my interpretations of these "fractal dimensions"). Ostwald et al. (2015) repeated the analysis using a more thorough analysis and reduced (but did not eliminate) the difference between the fractal dimensions of the two works, and a similar analysis performed by Wen and Kao (2005) were also able to discriminate the fractal dimensions of works by Frank Lloyd Wright, Le Corbusier and Mies van der Rohe. When applied to an architectural composition, the interpretation of the fractal dimension is that it is a measure of the detail at different scales, with a higher dimension revealing a higher level of detail. As Bovill (1996) put it: "The fractal characteristic of an architectural composition presents itself in this progression of interesting detail as one approaches, enters, and uses a building."

However, I should add the caveat that I do not think the front elevation of the above buildings is fractal (not even in an approximate way) for the following two reasons. First, authors used the box-counting method, and this method usually leads to a very small scaling region and very unreliable estimations of fractals characteristics, as discussed before. Second, and more importantly, I do not think the front elevation of these building have enough details at a range of scales that allows them to be called fractal. Nevertheless, these are just minor technical details, and I still found interesting, and important, to observe that techniques developed to analyse fractals can help quantify features that somehow agree with our intuitive perception of the designs.

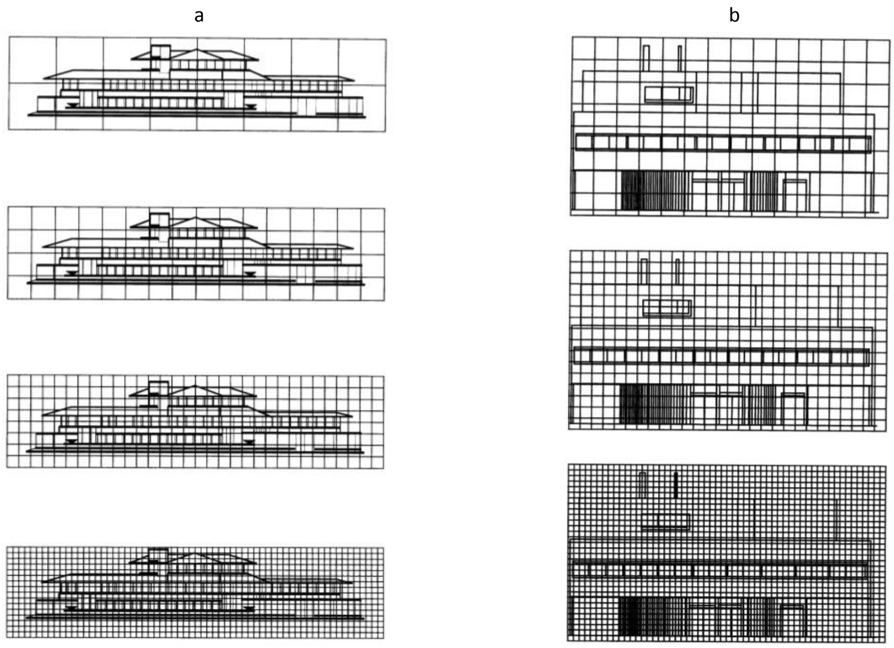

Figure 7. Example of the estimation of the fractal dimension using the box-counting method of the front elevation of (a) Robie house (Frank Lloyd Wright) and of (b) the Villa Savoye (Le Corbusier). Bovill (1996) found more than one scaling region, but fractal dimensions of the Robie House were consistently larger than those the Villa Savoye. [After Bovill (1996)]

5. Fractals and the critics of “modern” architecture

Somehow implicit in the research on music and bird songs mentioned above is that humans have an innate tendency to copy patterns that are present in nature and that, thanks to fractals, patterns that were being reproduced unconsciously are now being uncovered and identified by science (e.g., Voss & Clarke, 1975, 1978; Hsü, 1993, Van Tonder *et al*., 2002).

The relationship between the environment where our species evolved and some built-in preferences, that at a deeper level may be independent of an individual culture, was put forcible by Dutton (2009). I believe that such hypothesis of universals on human aesthetic preferences, undoubtedly controversial, provides a link between topics often kept in the realm of the humanities and of the natural sciences, with fractals providing a methodological approach to attain such a link. Indeed, the universality of aesthetic preferences is a recurrent theme in articles and books on fractals. For instance, Dauphiné (2012) states that

> When a group of people are asked to classify landscape images according to their beauty, responses show that regular, homogeneous landscapes and highly irregular landscapes are never picked out as being the most beautiful. People who are constantly questioned attribute great beauty to landscapes with intermediate irregularity. This mix of regularity, which is calm and secure, and

> irregularity, which is synonymous with anomaly and surprise, can be measured using a Hurst coefficient or a fractal dimension (Dauphiné, 2012, p.179).

Given that fractals are a paradigm of a mathematical approach that can reproduce faithfully natural objects, it is not surprising that fractals are mentioned in a criticism involving lack of naturalness, in particular about modern architecture (e.g., Alexander (2002) or Salingaros (1997, 2006)). Mandelbrot (1981) himself set the scene when he wrote that

> It is often said that 20th-century "modern" buildings are sterile, not built to human scale and, in fact, unnatural.

or in his famous book "The fractal geometry of nature" (Mandelbrot, 1983) when he stated that:

> The fractal "new geometric art" shows surprising kinship to Grand Master paintings or Beaux Arts architecture. An obvious reason is that classical visual arts, like fractals, involve very many scales of length and favor self similarity […]. For all these reasons, and also because it came in through an effort to imitate Nature in order to guess its laws, it may well be that fractal art is readily accepted because it is not true unfamiliar. Abstract paintings vary on this account: those I like tend to be close to fractal geometric art, but many are closer to standard geometric art - too close for my own comfort and enjoyment. […] A Mies van der Rohe building is a scale bound throwback to Euclid, while a high period Beaux Arts building is rich in fractal aspects.

The answer to the important question of why we should strive for fractal-like structures in architecture can probably be found in some intriguing studies on how human reacts to fractals (Taylor, 2006; Joye, 2007; and references therein), although these were not originally targeted at studying architectural features. For instance, in the 1980s NASA carried experiments to determine how to reduce stress in astronauts participating in long missions, such as, manned missions to Mars (Taylor, 2006). In these studies, participants were asked to performed several tasks, such as, arithmetic calculations, that are known to induce physiological stress, while observing different images. These experiments were conducted by measuring physiological parameters and not by assessing verbal or written preferences, therefore, using solely uncontrolled responses of the participants. The main result was that compared to a control non-fractal image, fractal images led to the reduction of stress levels. Moreover, researchers found that the reduction was higher for middle-range fractals, those that have a fractal dimension close to 1.5, that is, way from a smooth line (D=1) or a surface (D=2). As a justification to these results, Taylor (2006) points out that middle-range fractals are common in nature, such as in the contour of clouds or coastlines, indicating that this type of fractals are "a central feature of our daily visual experience". Notice, however, that in order to reduce stress the images did not need to be of natural objects or landscapes, they just needed to have fractal attributes that mimic those observed in natural objects, thus leaving space for a wide variety of forms, including abstract ones. The important point is "we have here the beginnings of a new way of interpreting how the visual environment affects our health" (Salingaros, 2012); and see also Sussman and Hollander (2015) or Ruggles (2017) where the authors explore the connections between architectural composition, how we perceive it, and neuroscience.

The previous paragraphs dealt with the absence, or reduction, of fractal attributes in some architectural compositions, and how that it can be detrimental to our health. However, I do not think that the application of fractals alone will ensure a sounded final architectural product. In fact, I have found buildings whose design is based on fractals that I can only describe as hideous. See, for instance, Fig. 8, a photo of a façade clearly inspired by the Sierpinski carpet (Fig. 1) but, in my opinion, with a rather poor result. Nor do I think that the inclusion of a fractal pattern as a decorative motif can alone be enough to bring the desire quality to the entire architectural composition. Consider, for instance, the fractal pattern based on the proportions of the golden rectangle suggested by Cecil Balmond to decorate a Daniel Libeskind's project in South Kensington, London (e.g., Langdon, 2015). According to Langdon, this fractal pattern would have "attempt[ed] to channel an energy that is cosmic and infinite". Unfortunately, I do not know what "energy that is cosmic" means, except for some esoteric (hence, pseudo-scientific) connotations, and I honestly do not see how such "infinite" form of energy would have benefited the workers, visitors and neighbors of the building, had it been built. Instead of this type of vague (and to me incomprehensible) language, I would rather have seen the results of tests similar to those described above conducted by NASA, where the stress levels of the participants were compared after performing a demanding task while facing the fractal of "infinite cosmic energy" and, say, the façade of other buildings in South Kensington.

In conclusion, although I think that the application of concepts from fractal geometry may be necessary for sounded architectural compositions, they are definitely not sufficient. For example, some basic respect for the traditions of a region and historical architecture of the already existing buildings seems to be a basic requirement. Other rules, such as the ones developed by Christopher (1977) (and see also Salingaros, 2006) are likely to be essential. Nor do I think that the adherence to strict formal fractal patterns, such as the Sierpinski carpet, is desired. If I had to identify the most valuable lesson from fractals to architecture, it would simply be *the importance of detail at different scales*.

6. Concluding remarks

This review had three main aims: first, to provide a brief description of different categories of fractals that are important to identify and characterize natural and man-made structures, second, to show that fractals are everywhere and, third, that fractal geometry is a tool that can unify disparate disciplines with important implications for the way we perceived and related to our environments.

Figure 8. A façade clearly inspired by the Sierpinski carpet (compare it to Fig. 1), but hardly a good example of application of fractals in architecture. [Photo by the author]

One of most intriguing aspects of fractal geometry is its ability to produces visual images that are astonishingly close to natural objects. In this respect, I would say that fractal geometry brought naturalness back to mathematics. Undoubtedly, mathematics has been a very effective tool in science and engineering, helping to describe natural phenomena (the motion of the planets in the solar system is a classic example) and to deliver real products (cars, airplanes, computers, radars, atomic bombs, etc.). However, to the lay person the connection between the abstract concepts of mathematics and real-world applications may not be immediately obvious. For instance, a brief excursion into the sections of a library on physics, or any other discipline that has been amenable to a rigorous mathematical description, will reveal books full of graphics with shapes that tend to be smooth, such as straight lines, circles, parabolas or hyperbolas. But these are not the curves of most natural objects, as Mandelbrot (1983) reminded us. On the other hand, any book on fractals will most likely (proudly) exhibit computer generated images that are difficult to distinguish from natural objects or landscapes. If those smooth curves in books on physics were at some point seen as a sign of progress and modernity, fractal geometry re-introduced "grainy, hydralike, in between, pimply, pocky, ramified, strange, tangled, tortuous, wiggly, wispy, wrinkled" shapes into contemporary scientific disciplines, and they may once again resurge as "modern".

The studies on how humans respond to fractals and non-fractal objects and designs have important implications to our well-being. So far, these studies have revealed that humans have an innate preference for fractals. Surely, different cultures have produced very different designs. However, the possible existence of unconscious universal preferences that are present in common patterns that can be reveal through fractals (or other methods) is tantalizing. Whether there are in fact such universals, and at which level, should not be taken uncritically, as recently showed about music by McDermott et al. (2016) (but see, e.g., Bowling *et al.*, 2017). It is, nevertheless, an interesting hypothesis that deserves further research and where fractal geometry may show the inherent consilience in human endeavours.

Acknowledgments

I thank Stephen P. Hubbell and Patricia A. Gowaty for several discussions on the connections between fractals and the real world, and I thank Malcolm Millais for several discussions on "Modern" architecture. This work was financed through Portuguese national funds through FCT – Fundação para a Ciência e a Tecnologia, I.P., under the Norma Transitória - DL57/2016/CP1440/CT0022.

References

Alexander, C. (1977). *A pattern language: towns, buildings, construction*. Oxford University Press, Oxford.

Alexander, C. (2002). *The nature of order: The phenomenon of life*. CA, Berkeley: The Center for Environmental Structure.

Barnsley, M.F. & Devaney, R.L. & Mandelbrot, B.B. & Peitgen, H.-O. & Saupe, D. & Voss, R.F. & Fisher, Y. & McGuire, M. (1988). *The Science of Fractal Images*. Springer, New York.

Batty, M., Longley, P.A. (1994). *Fractal cities: a geometry of form and function*. London. Academic press.

Barnsley, M. (1988). *Fractals everywhere: the first course in deterministic fractal geometry*. London. Academic Press.

Borda-de-Água, L.; Hubbell, S. P.; He, F. (2007). Scaling biodiversity under neutrality. In D. Storch, P.A. Marquet and J. H. Brown (Eds), *Scaling biodiversity* (pp. 347-375). Cambridge. Cambridge University Press.

Borda-de-Água, L. (2019). The Importance of Scaling in Biodiversity. In E. Casetta, J. M. Silva and D. Vecchi (Eds), *From Assessing to Conserving Biodiversity* (pp. 107-122). Springer, Cham.

Bovill, C. (1996). *Fractal geometry in architecture and design*. New York. Springer Science+Business Media.

Bowling, D.L., Hoeschele, M., Gill, K.Z., Fitch, W.T. (2017). The nature and nurture of musical consonance. *Music Perception: An Interdisciplinary Journal, 35*(1), 118-121.

Chen, Y. (2009). Analogies between urban hierarchies and river networks: Fractals, symmetry, and self-organized criticality. *Chaos, Solitons & Fractals, 40*(4), 1766-1778.

Dauphiné, A. (2013). *Fractal geography*. John Wiley & Sons.

Dotsenko, S. (2020). Fractals: Hunting for fractional dimension and attractive design. *New Design Ideas, 4*(1). 24-47.

Dutton, D. (2009). *The art instinct: Beauty, pleasure, and human evolution*. New York. Bloomsbury Press.
Holmes, A. (1965). *Principles of Physical Geology*, 2nd edition. Nelson, London.
Hsü, K.J. (1993). Fractal geometry of music: From bird songs to Bach. In A. J. Crilly, R. A. Earnshaw and H. Jones (Eds). *Applications of fractals and chaos* (pp. 21-39). Berlin. Springer.
Joye, Y. (2007). Architectural lessons from environmental psychology: the case of biophilic architecture. *Review of General Psychology*, *11*(4), 305-328.
Kim, D., Son, S.W., Jeong, H. (2014). Large-scale quantitative analysis of painting arts. *Scientific reports*, *4*, 7370.
Langdon, D. (2020). AD Classics: V&A Spiral / Daniel Lebeskind + Cecil Balmond. Arch Daily. Accessed July 21, https://www.archdaily.com/768565/ad-classics-v-and-a-spiral-daniel-libeskind-plus-cecil-balmond
Mandelbrot, B. (1975). *Les Objets Fractals*. Paris. Flammarion.
Mandelbrot, B. (1981). Scalebound or Scaling Shapes: A Useful Distinction in the Visual Arts and in the Natural Sciences. *Leonardo*, *14*(1), 45-47.
Mandelbrot, B.B. (1983). *The fractal geometry of nature*. New York. WH Freeman and Co.
McDermott, J.H., Schultz, A.F., Undurraga, E.A., Godoy, R.A. (2016). Indifference to dissonance in native Amazonians reveals cultural variation in music perception. *Nature*, *535*(7613), 547.
Ostwald, M.J., Vaughan, J., Tucker, C. (2015). Characteristic visual complexity: Fractal dimensions in the architecture of Frank Lloyd Wright and Le Corbusier. In K. Williams and M.J. Ostwald (Eds.), *Architecture and Mathematics from Antiquity to the Future* (pp. 339-354). Cham. Birkhäuser.
Peitgen, H.-O., Sauce, D. (1988). *The science of fractal images*. New York. Springer-Verlag.
Richardson, L.F. (1961). The problem of contiguity: an appendix to statistics of deadly quarrels. *General systems yearbook*, *6*, 139-187.
Ruggles, D. H. (2017). *Beauty, neuroscience and architecture: Timeless patterns and their impact on our well-being*. Fibonacci, LLC, Denver.
Salingaros, N.A. (1997). Life and complexity in architecture from a thermodynamic analogy. *Physics Essays*, *10*(1), 165-173.
Salingaros, N.A. (2006). *A theory of architecture*. UMBAU-VERLAG Harald Püschel.
Salingaros, N.A. (2012). Fractal art and architecture reduce physiological stress. *Journal of Biourbanism*, *II*(2), 2-12.
Sussman, A., & Hollander, J.B. (2015). *Cognitive Architecture*. Routledge, New York.
Taylor, R.P., Micolich, A.P., Jonas, D. (1999). Fractal analysis of Pollock's drip paintings. *Nature*, *399*(6735), 422.
Taylor, R.P. (2006). Reduction of physiological stress using fractal art and architecture. *Leonardo*, *39*(3), 245-251.
Van Tonder, G.J.; Lyons, M.J.; Ejima, Y. – Visual structure of a Japanese Zen garden. *Nature*, *419*(6905), 359. 2002
Voss, R.P., Clarke, J. (1975). '1/f noise' in music and speech. *Nature*, *258*(5533), 317-318.
Voss, R.F., Clarke, J. (1978). ''1/f noise'' in music: Music from 1/f noise. *The Journal of the Acoustical Society of America*, *63*(1), 258-263.
Wen, K.C., Kao, Y.N. (2005). An analytic study of architectural design style by fractal dimension method. In *22nd International Symposium on Automation and Robotics in Construction* (pp. 367-372).
Wilson, E.O. (1988). *Consilience: the unity of knowledge*. New York. Knopf.
Wong, D.W., Lasus, H., Falk, R.F. (1999). Exploring the variability of segregation index D with scale and zonal systems: an analysis of thirty US cities. *Environment and Planning A*, *31*(3), 507-522.